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Coupled Force and Moment Parameter Estimation for Aircraft

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This paper presents the full application of the multipoint aerodynamic model for parameter estimation in high-angle-of-attack and high-angular-rate flight regimes. This model is intended for use in simulation and control design, and for the study of aerodynamic force distributions on the surfaces of an aircraft. The estimation technique used to illustrate the approach is simple regression with the equation error approach. The multipoint model comprises a set of new parameters describing the aerodynamic force distribution along individual surface components of the aircraft. The aim of this study is to demonstrate that this model allows coupling among the three force and three moment components, which is accounted for in the model. This means that the parameters associated with the six-component equations are thus treated simultaneously. Another advantage of this approach is that the model allows each individual force-generating surface element of the aircraft to contribute independently to the total force and moment, rather than some average of these contributions relative to the c.m. The method is applied to measurements from spin flight test data conducted with a light general aviation aircraft. The results indicate that the method is capable of reproducing, with reasonable accuracy, the force and moment measurements obtained from a flight other than the one used in the parameter estimation.

Nomenclature

b	= span
C	= nondimensional aerodynamic coefficient or stability derivative
C_i	= force parameter or coefficient
d	= distance
F	= generic force component
$\boldsymbol{F}$	= force vector
$\boldsymbol{h}, \boldsymbol{k}$	= factors depending on the kinematic state at time t
L	= rolling moment
M	= generic moment component or pitching moment
N	= yawing moment or normal component
P, p	= roll rate, nondimensional roll rate
Q, q	= pitch rate, nondimensional pitch rate
$\tilde{q}$	= dynamic pressure
R, r	= yaw rate, nondimensional yaw rate
S	= area of an aircraft element
$\boldsymbol{u}$	= control vector
V	= airspeed
X	= force component along x axis
x	= moment arm in the x-axis direction
$\boldsymbol{x}$	= state vector
Y	= force component along y axis
y	= moment arm in the y-axis direction
Z	= force component along z axis
z	= moment arm in the z-axis direction
α	= angle of attack
β	= sideslip angle
δ_A	= aileron deflection
δ_E	= elevator deflection
δ_R	= rudder deflection
δ_T	= thrust setting, rpm
η	= spanwise parameter

Introduction

ESTABLISHED parameter estimation methods are being used successfully to determine the aerodynamic characteristics of an aircraft configuration from flight test measurements. These measurements consist of the usual motion variables, namely the components, in a set of body-fixed axes, of the linear and angular velocities. The former are equivalently available as velocity magnitude, angles of attack, and sideslip. Along with these, linear accelerometer readings and manipulations of the angular data are converted into aerodynamic force and moment components along the body axes. Two fundamental methods are employed to estimate the desired aerodynamic parameters. The equation error method uses a postulated model for these forces and moments, i.e., the assumed functional form for their dependence on the motion variables that generate them, and seeks the estimates of the unknown coefficients in the postulated model, by minimizing the error between the measurement and the corresponding value calculated on the basis of the estimate (regression analysis). The output error method numerically solves the equations of motion based on these estimates, and compares the resulting computed state to the measured one.

Current usage of these methods[1] deals with the coefficients representing the aerodynamics of the aircraft as a whole. This seems to work well for flight regimes moderately nonlinear in nature, but fails to provide general and robust results for the aerodynamics of maneuvers such as the spin, which is highly nonlinear, because of the large angles of attack and angular rates. During such motions, the behavior of one surface element can be radically different from that of another. In the current usage, these forces are lumped at the c.m. into one force, their sum, and hence one cannot expect that the regression could predict the behavior for one spin with data collected from another. The resulting model is tailored to the specific data used for its computation, thus requiring data partitioning into ranges of the independent variables.

The multipoint model seems to address this problem. In Refs. 2–5, this fact was demonstrated with a partial application

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of the method using simplified models. The work presented here is a more complete application of this modeling concept.

Description of the Aerodynamic Models

Conventional Model

The aerodynamic model used currently for parameter estimation is composed of equations for the forces and moments as functions of a specified set of state variables and controls. These equations have unknown parameters often in the form of partial derivatives to the state variables and controls. These are the parameters to be estimated.

The state vector comprises the airspeed, angles of attack and sideslip defined at the c.m., and the components of angular velocity. The control vector contains the control surface deflections and the thrust

$$\begin{aligned} \boldsymbol{x} &= \{V, \alpha, \beta, P, Q, R\}^T \\ \boldsymbol{u} &= \{\delta_A, \delta_E, \delta_R, \delta_T\}^T \end{aligned} \tag{1}$$

A functional relationship is stipulated. The rolling moment, for example, might be assumed to obey the following relationship:

$$L = L(\beta, P, R, \delta_A, \delta_R) \tag{2}$$

which can be expanded in a Taylor's series about some steady-state or reference condition $\boldsymbol{x}_0$, $\boldsymbol{u}_0$, and truncated at a suitable order

$$\begin{aligned} L(\boldsymbol{x}, \boldsymbol{u}) = L(\boldsymbol{x}_0, \boldsymbol{u}_0) &+ \frac{\partial L}{\partial \beta}\Delta\beta + \frac{\partial L}{\partial P}\Delta P + \frac{\partial L}{\partial R}\Delta R \\ &+ \frac{\partial L}{\partial \delta_A}\Delta\delta_A + \frac{\partial L}{\partial \delta_R}\Delta\delta_R + \frac{1}{2}\frac{\partial^2 L}{\partial \beta^2}(\Delta\beta)^2 \\ &+ \frac{1}{2}\frac{\partial^2 L}{\partial P^2}(\Delta P)^2 + \cdots \end{aligned} \tag{3}$$

Multipoint Model

The model used for the present work is based on strip theory. Because of the angular velocity, the dynamic pressure is a function of position on the surface of the aircraft. If η is introduced, upon which the local dynamic pressure and the angles of attack and sideslip are dependent, then $\boldsymbol{x}$ in Eq. (1) can be replaced by their derivatives with respect to η as shown in the following text.

Force equation: because

$$F = \int_0^b F(\eta)\,\mathrm{d}\eta = \int_0^b \tilde{q}(\eta)C(\eta)c(\eta)\,\mathrm{d}\eta$$

where

$$\tilde{q}(\eta) = \tilde{q} + \frac{\mathrm{d}\tilde{q}}{\mathrm{d}\eta}\eta + \frac{1}{2}\frac{\mathrm{d}^2\tilde{q}}{\mathrm{d}\eta^2}\eta^2, \quad C(\eta) = C + \frac{\mathrm{d}C}{\mathrm{d}\eta}\eta + \frac{1}{2}\frac{\mathrm{d}^2 C}{\mathrm{d}\eta^2}\eta^2$$

$$+ \cdots, \quad \text{and} \quad c(\eta) = \bar{c}, \text{ assumed to be constant}$$

then

$$\begin{aligned} F = \int_0^b &\left\{\tilde{q} + \frac{\mathrm{d}\tilde{q}}{\mathrm{d}\eta}\eta + \frac{1}{2}\frac{\mathrm{d}^2\tilde{q}}{\mathrm{d}\eta^2}\eta^2\right\} \\ &\times \left\{C + \frac{\mathrm{d}C}{\mathrm{d}\eta}\eta + \frac{1}{2}\frac{\mathrm{d}^2 C}{\mathrm{d}\eta^2}\eta^2\right\}\cdot\bar{c}\,\mathrm{d}\eta \end{aligned}$$

Because

$$C = f[\alpha(\eta)], \qquad \frac{\mathrm{d}C}{\mathrm{d}\eta} = \frac{\mathrm{d}C}{\mathrm{d}\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\eta}$$

$$\frac{\mathrm{d}^2C}{\mathrm{d}\eta^2} = \frac{\mathrm{d}C}{\mathrm{d}\alpha}\frac{\mathrm{d}^2\alpha}{\mathrm{d}\eta^2} + \frac{\mathrm{d}^2C}{\mathrm{d}\alpha^2}\left(\frac{\mathrm{d}\alpha}{\mathrm{d}\eta}\right)^2$$

$$\begin{aligned} F = \int_0^b &\left[\tilde{q} + \frac{\mathrm{d}\tilde{q}}{\mathrm{d}\eta}\eta + \frac{1}{2}\frac{\mathrm{d}^2\tilde{q}}{\mathrm{d}\eta^2}\eta^2\right]\cdot\left[C + \frac{\mathrm{d}C}{\mathrm{d}\alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}\eta}\eta\right. \\ &\left. + \frac{1}{2}\left\{\frac{\mathrm{d}C}{\mathrm{d}\alpha}\frac{\mathrm{d}^2\alpha}{\mathrm{d}\eta^2} + \frac{\mathrm{d}^2C}{\mathrm{d}\alpha^2}\left(\frac{\mathrm{d}\alpha}{\mathrm{d}\eta}\right)^2\right\}\eta^2\right]\cdot\bar{c}\,\mathrm{d}\eta \end{aligned}$$

define

$$\delta\tilde{q} = \frac{\mathrm{d}\tilde{q}}{\mathrm{d}\eta}, \qquad \delta\alpha = \frac{\mathrm{d}\alpha}{\mathrm{d}\eta}, \qquad \mathrm{d}C = \frac{\mathrm{d}C}{\mathrm{d}\alpha}, \qquad \delta^2\alpha = \frac{1}{2}\frac{\mathrm{d}^2\alpha}{\mathrm{d}\eta^2}$$

$$\delta^2\tilde{q} = \frac{1}{2}\frac{\mathrm{d}^2\tilde{q}}{\mathrm{d}\eta^2}, \qquad \mathrm{d}^2C = \frac{1}{2}\frac{\mathrm{d}^2C}{\mathrm{d}\alpha^2}$$

then

$$\begin{aligned} F(\eta) = {} &[\tilde{q} + \delta\tilde{q}\cdot\eta + \delta^2\tilde{q}\cdot\eta^2]\cdot[C + \mathrm{d}C\cdot\delta\alpha\cdot\eta \\ &+ \mathrm{d}C\cdot\delta^2\alpha\cdot\eta^2 + \mathrm{d}^2C\cdot(\delta\alpha)^2\cdot\eta^2]\cdot\bar{c} \end{aligned} \tag{4}$$

Assuming $\boldsymbol{C}$ to be related to the local angle of attack according to Eq. (4), then

$$C(\alpha) = C_1 + C_2\alpha + C_3\alpha^2 + C_4\alpha^3 + C_5\alpha^4$$

Substituting in Eq. (4) and rearranging

$$F(\eta) = [a_0 + a_1\cdot\eta + a_2\cdot\eta^2 + a_3\cdot\eta^3 + a_4\cdot\eta^4]\cdot\bar{c} \tag{5}$$

where

$$\begin{aligned}
a_0 = {} & C_1\cdot\tilde{q} + C_2\cdot\tilde{q}\cdot\alpha + C_3\cdot\tilde{q}\cdot\alpha^2 + C_4\cdot\tilde{q}\cdot\alpha^3 + C_5\cdot\tilde{q}\cdot\alpha^4 \\
a_1 = {} & C_1\cdot\delta\tilde{q} + C_2\cdot\{\tilde{q}\cdot\delta\alpha + \delta\tilde{q}\cdot\alpha\} \\
& + C_3\cdot\{2\tilde{q}\cdot\delta\alpha\cdot\alpha + \delta\tilde{q}\cdot\alpha^2\} + C_4\cdot\{3\tilde{q}\cdot\delta\alpha\cdot\alpha^2 + \delta\tilde{q}\cdot\alpha^3\} \\
& + C_5\cdot\{4\tilde{q}\cdot\delta\alpha\cdot\alpha^3 + \delta\tilde{q}\cdot\alpha^4\} \\
a_2 = {} & C_1\cdot\delta^2\tilde{q} + C_2\cdot\{\tilde{q}\cdot\delta^2\alpha + \delta^2\tilde{q}\cdot\alpha + \delta\tilde{q}\cdot\delta\alpha\} \\
& + C_3\cdot\{2\tilde{q}\cdot\delta^2\alpha\cdot\alpha + \delta^2\tilde{q}\cdot\alpha^2 + 2\delta\tilde{q}\cdot\delta\alpha\cdot\alpha + \tilde{q}\cdot(\delta\alpha)^2\} \\
& + C_4\cdot\{3\tilde{q}\cdot\delta^2\alpha\cdot\alpha^2 + \delta^2\tilde{q}\cdot\alpha^3 + 3\delta\tilde{q}\cdot\delta\alpha\cdot\alpha^2 \\
& + 3\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha\} \\
& + C_5\cdot\{4\tilde{q}\cdot\delta^2\alpha\cdot\alpha^3 + \delta^2\tilde{q}\cdot\alpha^4 + 4\delta\tilde{q}\cdot\delta\alpha\cdot\alpha^3 \\
& + 6\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha^2\} \\
a_3 = {} & C_2\cdot\{\delta\tilde{q}\cdot\delta^2\alpha + \delta^2\tilde{q}\cdot\delta\alpha\} \\
& + C_3\cdot\{2\delta\tilde{q}\cdot\delta^2\alpha\cdot\alpha + 2\delta^2\tilde{q}\cdot\delta\alpha\cdot\alpha + \delta q\cdot(\delta\alpha)^2\} \\
& + C_4\cdot\{3\delta\tilde{q}\cdot\delta^2\alpha\cdot\alpha^2 + 3\delta^2\tilde{q}\cdot\delta\alpha\cdot\alpha^2 + 3\delta\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha\} \\
& + C_5\cdot\{4\delta\tilde{q}\cdot\delta^2\alpha\cdot\alpha^3 + 4\delta^2\tilde{q}\cdot\delta\alpha\cdot\alpha^3 + 6\delta\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha^2\} \\
a_4 = {} & C_2\cdot\delta^2\tilde{q}\cdot\delta^2\alpha + C_3\cdot\{2\delta^2\tilde{q}\cdot\delta^2\alpha\cdot\alpha + \delta^2 q\cdot(\delta\alpha)^2\} \\
& + C_4\cdot\{3\delta^2\tilde{q}\cdot\delta^2\alpha\cdot\alpha^2 + 3\delta^2\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha\} \\
& + C_5\cdot\{4\delta^2\tilde{q}\cdot\delta^2\alpha\cdot\alpha^3 + 6\delta^2\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha^2\}
\end{aligned}$$

A generic force component results from integrating the distribution $F(\eta)$ along the line of reference as follows:

$$F = \int_0^b F(\eta)\,\mathrm{d}\eta = [C_1\cdot k_1 + C_2\cdot k_2 + C_3\cdot k_3 + C_4\cdot k_4 + C_5\cdot k_5]\cdot\bar{c}$$

where the k_i coefficients depend strictly on the kinematic state of the aircraft and the air density, so that

$$k_1 = \tilde{q}\cdot b + \tfrac{1}{2}\delta\tilde{q}\cdot b^2 + \tfrac{1}{3}\delta^2\tilde{q}\cdot b^3$$

$$\begin{aligned}
k_2 = {} & \tilde{q}\cdot\alpha\cdot b + \{\tfrac{1}{2}\tilde{q}\cdot\delta\alpha + \tfrac{1}{2}\delta\tilde{q}\cdot\alpha\}\cdot b^2 \\
& + \{\tfrac{1}{3}\tilde{q}\cdot\delta^2\alpha + \tfrac{1}{3}\delta^2\tilde{q}\cdot\alpha + \tfrac{1}{3}\delta\tilde{q}\cdot\delta\alpha\}\cdot b^3 \\
& + \{\tfrac{1}{4}\delta\tilde{q}\cdot\delta^2\alpha + \tfrac{1}{4}\delta^2\tilde{q}\cdot\delta\alpha\}\cdot b^4 + \tfrac{1}{5}\delta^2\tilde{q}\cdot\delta^2\alpha\cdot b^5
\end{aligned}$$

$$\begin{aligned}
k_3 = {} & \tilde{q}\cdot\alpha^2\cdot b + \{\tilde{q}\cdot\delta\alpha\cdot\alpha + \tfrac{1}{2}\delta\tilde{q}\cdot\alpha^2\}\cdot b^2 \\
& + \{\tfrac{1}{3}\tilde{q}\cdot(\delta\alpha)^2 + \tfrac{2}{3}\tilde{q}\cdot\delta^2\alpha\cdot\alpha + \tfrac{1}{3}\delta^2\tilde{q}\cdot\alpha^2 \\
& + \tfrac{2}{3}\delta q\cdot\delta\alpha\cdot\alpha\}\cdot b^3 + \{\tfrac{1}{2}\delta\tilde{q}\cdot\delta^2\alpha\cdot\alpha \\
& + \tfrac{1}{4}\delta\tilde{q}\cdot(\delta\alpha)^2 + \tfrac{1}{2}\delta^2\tilde{q}\cdot\delta\alpha\cdot\alpha\}\cdot b^4 \\
& + \{\tfrac{2}{5}\delta^2\tilde{q}\cdot\delta^2\alpha\cdot\alpha + \tfrac{1}{5}\delta^2\tilde{q}\cdot(\delta\alpha)^2\}\cdot b^5
\end{aligned}$$

$$\begin{aligned}
k_4 = {} & \tilde{q}\cdot\alpha^3\cdot b + \{\tfrac{3}{2}\tilde{q}\cdot\delta\alpha\cdot\alpha^2 \\
& + \tfrac{1}{2}\delta\tilde{q}\cdot\alpha^3\}\cdot b^2 + \{\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha + \tilde{q}\cdot\delta^2\alpha\cdot\alpha^2 \\
& + \tfrac{1}{3}\delta^2\tilde{q}\cdot\alpha^3 + \delta\tilde{q}\cdot\delta\alpha\cdot\alpha^2\}\cdot b^3 \\
& + \{\tfrac{3}{4}\delta\tilde{q}\cdot\delta^2\alpha\cdot\alpha^2 + \tfrac{3}{4}\delta\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha \\
& + \tfrac{3}{4}\delta^2\tilde{q}\cdot\delta\alpha\cdot\alpha^2\}\cdot b^4 + \{\tfrac{3}{5}\delta^2\tilde{q}\cdot\delta^2\alpha\cdot\alpha^2 \\
& + \tfrac{3}{5}\delta^2\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha\}\cdot b^5
\end{aligned}$$

$$\begin{aligned}
k_5 = {} & \tilde{q}\cdot\alpha^4\cdot b + \{2\tilde{q}\cdot\delta\alpha\cdot\alpha^3 + \tfrac{1}{2}\delta\tilde{q}\cdot\alpha^4\}\cdot b^2 \\
& + \{2\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha^2 + \tfrac{4}{3}\tilde{q}\cdot\delta^2\alpha\cdot\alpha^3 \\
& + \tfrac{1}{3}\delta^2\tilde{q}\alpha^4 + \tfrac{4}{3}\delta\tilde{q}\cdot\delta\alpha\cdot\alpha^3\}\cdot b^3 \\
& + \{\delta\tilde{q}\cdot\delta^2\alpha\cdot\alpha^3 + \tfrac{3}{2}\delta\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha^2 + \delta^2\tilde{q}\cdot\delta\alpha\cdot\alpha^3\}\cdot b^4 \\
& + \{\tfrac{4}{5}\delta^2\tilde{q}\cdot\delta^2\alpha\cdot\alpha^3 + \tfrac{6}{5}\delta^2\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha^2\}\cdot b^5
\end{aligned}$$

The moment generated by this same force distribution is simply

$$\begin{aligned}
M = {} & \int_0^b F(\eta)\cdot\eta\ d\eta = [C_1\cdot h_1 + C_2\cdot h_2 + C_3\cdot h_3 \\
& + C_4\cdot h_4 + C_5\cdot h_5]\cdot\bar{c}
\end{aligned}$$

where the h_i are coefficients similar to k_i

$$h_1 = \tfrac{1}{2}\tilde{q}\cdot b^2 + \tfrac{1}{3}\delta\tilde{q}\cdot b^3 + \tfrac{1}{4}\delta^2\tilde{q}\cdot b^4$$

$$\begin{aligned}
h_2 = {} & \tfrac{1}{2}\tilde{q}\cdot\alpha\cdot b^2 + \{\tfrac{1}{3}\delta\tilde{q}\cdot\alpha + \tfrac{1}{3}\tilde{q}\cdot\delta\alpha\}\cdot b^3 \\
& + \{\tfrac{1}{4}\delta^2\tilde{q}\cdot\alpha + \tfrac{1}{4}\tilde{q}\cdot\delta^2\alpha + \tfrac{1}{4}\delta\tilde{q}\cdot\delta\alpha\}\cdot b^4 \\
& + \{\tfrac{1}{5}\delta\tilde{q}\cdot\delta^2\alpha + \tfrac{1}{5}\delta^2\tilde{q}\cdot\delta\alpha\}\cdot b^5 + \tfrac{1}{6}\delta^2\tilde{q}\cdot\delta^2\alpha\cdot b^6
\end{aligned}$$

$$\begin{aligned}
h_3 = {} & \tfrac{1}{2}\tilde{q}\cdot\alpha^2\cdot b^2 + \{\tfrac{1}{3}\delta\tilde{q}\cdot\alpha^2 + \tfrac{2}{3}\tilde{q}\cdot\delta\alpha\cdot\alpha\}\cdot b^3 \\
& + \{\tfrac{1}{4}\tilde{q}\cdot(\delta\alpha)^2 + \tfrac{1}{4}\delta^2\tilde{q}\cdot\alpha^2 + \tfrac{1}{2}\tilde{q}\cdot\delta^2\alpha\cdot\alpha \\
& + \tfrac{1}{2}\delta\tilde{q}\cdot\delta\alpha\cdot\alpha\}\cdot b^4 + \{\tfrac{1}{5}\delta\tilde{q}\cdot(\delta\alpha)^2 \\
& + \tfrac{2}{5}\delta\tilde{q}\cdot\delta^2\alpha\cdot\alpha + \tfrac{2}{5}\delta^2\tilde{q}\cdot\delta\alpha\cdot\alpha\}\cdot b^5 \\
& + \{\tfrac{1}{3}\delta^2\tilde{q}\cdot\delta^2\alpha\cdot\alpha + \tfrac{1}{6}\delta^2\tilde{q}\cdot(\delta\alpha)^2\}\cdot b^6
\end{aligned}$$

$$\begin{aligned}
h_4 = {} & \tfrac{1}{2}\tilde{q}\cdot\alpha^3\cdot b^2 + \{\tfrac{1}{3}\delta\tilde{q}\cdot\alpha^3 + \tilde{q}\cdot\delta\alpha\cdot\alpha^2\}\cdot b^3 \\
& + \{\tfrac{3}{4}\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha + \tfrac{1}{4}\delta^2\tilde{q}\cdot\alpha^3 + \tfrac{3}{4}\tilde{q}\cdot\delta^2\alpha\cdot\alpha^2 \\
& + \tfrac{3}{4}\delta\tilde{q}\cdot\delta\alpha\cdot\alpha^2\}\cdot b^4 + \{\tfrac{3}{5}\delta\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha \\
& + \tfrac{3}{5}\delta\tilde{q}\cdot\delta^2\alpha\cdot\alpha^2 + \tfrac{3}{5}\delta^2\tilde{q}\cdot\delta\alpha\cdot\alpha^2\}\cdot b^5 \\
& + \{\tfrac{1}{2}\delta^2\tilde{q}\cdot\delta^2\alpha\cdot\alpha^2 + \tfrac{1}{2}\delta^2\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha\}\cdot b^6
\end{aligned}$$

$$\begin{aligned}
h_5 = {} & \tfrac{1}{2}\tilde{q}\cdot\alpha^4\cdot b^2 + \{\tfrac{1}{3}\delta\tilde{q}\cdot\alpha^4 \\
& + \tfrac{4}{3}\tilde{q}\cdot\delta\alpha\cdot\alpha^3\}\cdot b^3 + \{\tfrac{3}{2}\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha^2 \\
& + \tfrac{1}{4}\delta^2\tilde{q}\cdot\alpha^4 + \tilde{q}\cdot\delta^2\alpha\cdot\alpha^3 + \delta\tilde{q}\cdot\delta\alpha\cdot\alpha^3\}\cdot b^4 \\
& + \{\tfrac{6}{5}\delta\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha^2 + \tfrac{4}{5}\delta\tilde{q}\cdot\delta^2\alpha\cdot\alpha^3 \\
& + \tfrac{4}{5}\delta^2\tilde{q}\cdot\delta\alpha\cdot\alpha^3\}\cdot b^5 + \{\tfrac{2}{3}\delta^2\tilde{q}\cdot\delta^2\alpha\cdot\alpha^3 \\
& + \delta^2\tilde{q}\cdot(\delta\alpha)^2\cdot\alpha^2\}\cdot b^6
\end{aligned}$$

Note that $\boldsymbol{k}$ and $\boldsymbol{h}$ are kinematic factors dependent on the dynamic pressure and angle-of-attack distribution along η, thus they are known quantities for each measurement. They are multiplied by $\cos^2\beta$ to account for sideslip according to the $\cos^2\beta$ law. It is important to note that the same coefficients C appear in both force and moment equations. We proceed next to develop these equations for each contributing surface point, transform them to the common aircraft body–axis system, and finally produce the force and moment equations for the entire aircraft. Some simplifying assumptions are introduced, namely that the airflow is steady and that the effects of the wing and fuselage on the remaining surfaces are not directly accounted for. The regression is expected to account for these effects indirectly, thus introducing some approximation.

Development of the Model for Regression

Table 1 lists the surface elements (points) and the force components to which they contribute. These force components cause the corresponding moment components shown in the third column.

We start with the axial and normal force coefficients C_A and C_N for the aerodynamic surface points and the engine thrust model. The most general model consists of five parameters, as in those used for the wing (Table 1). These parameters, to be estimated in the regression process, are the coefficients in the power series of the local angle of attack. To reduce the number of unknown parameters, it is necessary to choose the minimum number of parameters to adequately model any of the surface points. For example, the aileron C_N has an assumed linear dependence on angle of attack, because it is not exposed to the wing/fuselage wake. This is unlike the vertical stabilizer, for which a cubic relationship is assumed.

Wing:

$$C_{A_W} = C_1 + C_2\cdot\alpha + C_3\cdot\alpha^2 + C_4\cdot\alpha^3 + C_5\cdot\alpha^4$$

$$C_{N_W} = C_{23} + C_{24}\cdot\alpha + C_{25}\cdot\alpha^2 + C_{26}\cdot\alpha^3 + C_{27}\cdot\alpha^4$$

Aileron:

$$C_{A_{\text{ail}}} = C_6 + C_7\cdot\alpha + C_8\cdot\alpha^2$$

$$C_{N_{\text{ail}}} = C_{28}\cdot\alpha$$

Table 1 Force and moment components due to surface points

Surface point	Force component	Moment component
Right wing/left wing	X	N, M
	Y	L
	Z	L, M
Fuselage	Y	N
Horizontal stabilizer	X	N
	Z	L, M
Vertical stabilizer	X	——
	Y	L, N
Thrust	X	——
	Y	N
Elevator	X	——
	Z	M
Rudder	X	——
	Y	L, N
Aileron	X	N
	Z	L

Horizontal stabilizer:

$$C_{A_{ht}} = C_9 + C_{10}\cdot\alpha + C_{11}\cdot\alpha^2 + C_{12}\cdot\alpha^3$$
$$C_{N_{ht}} = C_{29} + C_{30}\cdot\alpha + C_{31}\cdot\alpha^2 + C_{32}\cdot\alpha^3$$

Elevator:

$$C_{A_{ele}} = C_{13} + C_{14}\cdot\alpha + C_{15}\cdot\alpha^2$$
$$C_{N_{ele}} = C_{33}\cdot\alpha$$

Vertical stabilizer:

$$C_{A_{vt}} = C_{16} + C_{17}\cdot\alpha + C_{18}\cdot\alpha^2 + C_{19}\cdot\alpha^3$$
$$C_{N_{vt}} = C_{34} + C_{35}\cdot\alpha + C_{36}\cdot\alpha^2 + C_{37}\cdot\alpha^3$$

Rudder:

$$C_{A_{rud}} = C_{20} + C_{21}\cdot\alpha + C_{22}\cdot\alpha^2$$
$$C_{N_{rud}} = C_{38}\cdot\alpha$$

Fuselage:

$$C_{N_{fu}} = C_{39} + C_{40}\cdot\alpha + C_{41}\cdot\alpha^2 + C_{42}\cdot\alpha^3$$

Engine:

$$R_\rho = \rho_h/\rho_0$$
$$P_h = P_0\{R_\rho - [(1 - R_\rho)/7.55]\}$$
$$P_A = P_h(\omega_p/\omega_{max})^3$$
$$J_{75} = U_p/(\omega_p R_{75})$$
$$R_J = J_{75}/J_{max}$$
$$\text{eff} = 4\ \text{eff}_{max} R_J(1 - R_j)$$
$$F_X \approx \chi_{prop} = 550[\text{eff}\cdot P_A)/U_p]$$

In terms of the kinematic quantities $\boldsymbol{k}$ and $\boldsymbol{h}$, the surface point contributions are given next as axial and normal forces, then resolved along the body (x, y, z) axes. Also, their moments about the same axes are computed. The transformations involve only the surface incidence angle i and anhedral angle Γ because, in this example, there are no sweep angles.

Wing:

Axial and normal forces:

$$F_A = [C_1\cdot k_1 + C_2\cdot k_2 + C_3\cdot k_3 + C_4\cdot k_4 + C_5\cdot k_5]\cdot \bar{c}_W$$
$$F_N = [C_{23}\cdot k_{23} + C_{24}\cdot k_{24} + C_{25}\cdot k_{25} + C_{26}\cdot k_{26} + C_{27}\cdot k_{27}]\cdot \bar{c}_W$$

x-direction force F_x:

$$F_{X_W} = -F_A\cdot\cos(i_W) - F_N\cdot\sin(i_W)$$
$$F_{X_W} = F^R_{X_W} + F^L_{X_W}$$

y-direction force F_Y:

$$F_{Y_W} = [F_N\cdot\cos(i_W) - F_A\cdot\sin(i_W)]\cdot\sin(\Gamma)$$
$$F_{Y_W} = F^R_{Y_W} - F^L_{Y_W}$$

z-direction force F_Z:

$$F_{Z_W} = -[F_N\cdot\cos(i_W) - F_A\cdot\sin(i_W)]\cdot\cos(\Gamma)$$
$$F_{Z_W} = F^R_{Z_W} + F^L_{Z_W}$$

Rolling moment L:

$$L_W = \int_0^b [F_N(\eta)\cdot\cos(i_W) - F_A(\eta)\cdot\sin(i_W)]\cdot\eta\ d\eta + F_{Y_W}\cdot dz_W$$
$$= \{[C_{23}\cdot h_{23} + C_{24}\cdot h_{24} + C_{25}\cdot h_{25} + C_{26}\cdot h_{26}$$
$$+ C_{27}\cdot h_{27}]\cos(i_W) - [C_1\cdot h_1 + C_2\cdot h_2 + C_3\cdot h_3 + C_4\cdot h_4$$
$$+ C_5\cdot h_5]\cdot\sin(i_W)\}\cdot\bar{c}_W\cdot\cos(\Gamma) + F_{Y_W}\cdot dz_W$$
$$L_W = L^L_W - L^R_W$$

Pitching moment M:

$$M_W = [F_N\cdot\cos(i_W) - F_A\cdot\sin(i_W)]\cdot\cos(\Gamma)\cdot dx_W - F_{X_W}\cdot dz_W$$
$$M_W = M^R_W + M^L_W$$

Yawing moment N:

$$N_W = \int_0^b [F_A(\eta)\cdot\cos(i_W) + F_N(\eta)\cdot\sin(i_W)]\cdot\eta\ d\eta$$
$$= [C_1\cdot h_1 + C_2\cdot h_2 + C_3\cdot h_3 + C_4\cdot h_4$$
$$+ C_5\cdot h_5]\cdot\cos(i_W)\cdot\bar{c}_W + [C_{23}\cdot h_{23} + C_{24}\cdot h_{24} + C_{25}\cdot h_{25}$$
$$+ C_{26}\cdot h_{26} + C_{27}\cdot h_{27}]\cdot\sin(i_W)\cdot\bar{c}_W$$
$$N_W = N^R_W - N^L_W$$

Aileron:

Axial and normal forces:

$$F_A = [C_6\cdot k'_6 + C_7\cdot k'_7 + C_8\cdot k'_8]\cdot\bar{c}_{ail}$$
$$F_N = [C_{28}\cdot k'_{28}]\cdot\bar{c}_{ail}$$

x-direction force F_X:

$$F_{X_{ail}} = -F_A\cdot\cos(i_W + \delta a) - F_N\cdot\sin(i_W + \delta a)$$
$$F_{X_{ail}} = F^R_{X_{ail}} + F^L_{X_{ail}}$$

y-direction force F_Y:

$$F_{Y_{ail}} = [F_N\cdot\cos(i_W + \delta a) - F_A\cdot\sin(i_W + \delta a)]\cdot\sin(\Gamma)$$
$$F_{Y_{ail}} = F^R_{Y_{ail}} - F^L_{Y_{ail}}$$

z-direction force F_X:

$$F_{Z_{ail}} = [-F_N\cdot\cos(i_W + \delta a) + F_A\cdot\sin(i_W + \delta a)]\cdot\cos(\Gamma)$$
$$F_{Z_{ail}} = F^R_{Z_{ail}} + F^L_{Z_{ail}}$$

Rolling moment L:

$$L_{ail} = \{C_{28}\cdot h'_{28}\cdot\cos(i_W + \delta a) - [C_6\cdot h'_6 + C_7\cdot h'_7$$
$$+ C_8\cdot h'_8]\cdot\bar{c}_{ail}\cdot\sin(i_W + \delta a)\}\cdot\cos(\Gamma) + F_{Y_{ail}}\cdot dz_{ail}$$
$$L_{ail} = L^L_{ail} - L^R_{ail}$$

Pitching moment M:

$$M_{ail} = [F_N\cdot\cos(i_W + \delta a)$$
$$- F_A\cdot\sin(i_W + \delta a)]\cdot\cos(\Gamma)\cdot dx_{ail} - F_{X_{ail}}\cdot dz_{ail}$$
$$M_{ail} = M^R_{ail} + M^L_{ail}$$

Yawing moment N:

$$N_{ail} = \{[C_6\cdot h'_6 + C_7\cdot h'_7 + C_8\cdot h'_8]\cdot\cos(i_W + \delta a)$$
$$+ C_{28}\cdot h'_{28}\cdot\sin(i_W + \delta a)\}\cdot\bar{c}_{ail}$$
$$N_{ail} = N^R_{ail} - N^L_{ail}$$

Horizontal Stabilizer:
Axial and normal force:

$$F_A = [C_9 \cdot k_9 + C_{10} \cdot k_{10} + C_{11} \cdot k_{11} + C_{12} \cdot k_{12}] \cdot \bar{c}_{ht}$$

$$F_N = [C_{29} \cdot k_{29} + C_{30} \cdot k_{50} + C_{31} \cdot k_{31} + C_{32} \cdot k_{32}] \cdot \bar{c}_{ht}$$

x-direction force F_X:

$$F_{X_{ht}} = -F_A \cdot \cos(i_{ht}) - F_N \cdot \sin(i_{ht})$$

$$F_{X_{ht}} = F^R_{X_{ht}} + F^L_{X_{ht}}$$

z-direction force F_Z:

$$F_{Z_{ht}} = -F_N \cdot \cos(i_{ht}) + F_A \cdot \sin(i_{ht})$$

$$F_{Z_{ht}} = F^R_{Z_{ht}} + F^L_{Z_{ht}}$$

Pitching moment M:

$$M_{ht} = [F_N \cdot \cos(i_{ht}) - F_A \cdot \sin(i_{ht})] \cdot \mathrm{d}x_{ht} - F_{X_{ht}} \cdot dz_{ht}$$

$$M_{ht} = M^R_{ht} + M^L_{ht}$$

Elevator:
Axial and normal force:

$$F_A = [C_{13} \cdot k_{13} + C_{14} \cdot k_{14} + C_{15} \cdot k_{15}] \bar{c}_{\mathrm{ele}}$$

$$F_N = [C_{33} \cdot k_{33}] \cdot \bar{c}_{\mathrm{ele}}$$

x-direction force F_X:

$$F_{X_{\mathrm{ele}}} = -F_A \cdot \cos(i_{ht} + \delta e) - F_N \cdot \sin(i_{ht} + \delta e)$$

$$F_{X_{\mathrm{ele}}} = F^R_{X_{\mathrm{ele}}} + F^L_{X_{\mathrm{ele}}}$$

z-direction force F_Z:

$$F_{Z_{\mathrm{ele}}} = -F_N \cdot \cos(i_{ht} + \delta e) + F_A \cdot \sin(i_{ht} + \delta e)$$

$$F_{Z_{\mathrm{ele}}} = F^R_{Z_{\mathrm{ele}}} + F^L_{Z_{\mathrm{ele}}}$$

Pitching moment M:

$$M_{\mathrm{ele}} = [F_N \cdot \cos(i_{ht} + \delta e) - F_A \cdot \sin(i_{ht} + \delta e)] \cdot dx_{\mathrm{ele}} - F_{X_{\mathrm{ele}}} \cdot dz_{\mathrm{ele}}$$

$$M_{\mathrm{ele}} = M^R_{\mathrm{ele}} + M^L_{\mathrm{ele}}$$

Vertical Stabilizer:
x-direction force F_X:

$$F_{X_{\mathrm{vt}}} = -[C_{16} \cdot k_{16} + C_{17} \cdot k_{17} + C_{18} \cdot k_{18} + C_{19} \cdot k_{19}] \cdot \bar{c}_{\mathrm{vt}}$$

y-direction force F_Y:

$$F_{Y_{\mathrm{vt}}} = -[C_{34} \cdot k_{34} + C_{35} \cdot k_{35} + C_{36} \cdot k_{36} + C_{37} \cdot k_{37}] \cdot \bar{c}_{\mathrm{vt}}$$

Rolling moment L:

$$L_{\mathrm{vt}} = -[C_{34} \cdot h_{34} + C_{35} \cdot h_{35} + C_{36} \cdot h_{36} + C_{37} \cdot h_{37}] \cdot \bar{c}_{\mathrm{vt}}$$

Yawing moment N:

$$N_{\mathrm{vt}} = [C_{34} \cdot k_{34} + C_{35} \cdot k_{35} + C_{36} \cdot k_{36} + C_{37} \cdot k_{37}] \cdot \bar{c}_{\mathrm{vt}} \cdot dx_{\mathrm{vt}}$$

Rudder:
Axial and normal force:

$$F_A = [C_{20} \cdot k_{20} + C_{21} \cdot k_{21} + C_{22} \cdot k_{22}] \cdot \bar{c}_{\mathrm{rud}}$$

$$F_N = [C_{38} \cdot k_{38}] \cdot \bar{c}_{\mathrm{rud}}$$

x-direction force F_X:

$$F_{X_{\mathrm{rud}}} = -F_A \cdot \cos(-\delta r) - F_N \cdot \sin(-\delta r)$$

y-direction force F_Y:

$$F_{Y_{\mathrm{rud}}} = -F_N \cdot \cos(-\delta r) + F_A \cdot \sin(-\delta r)$$

Rolling moment L:

$$L_{\mathrm{rud}} = -C_{38} \cdot h_{38} \cdot \bar{c}_{\mathrm{rud}} \cdot \cos(-\delta r) + [C_{20} \cdot h_{20} + C_{21} \cdot h_{21} + C_{22} \cdot h_{22}] \cdot \bar{c}_{\mathrm{rud}} \cdot \sin(-\delta r)$$

Yawing moment N:

$$N_{\mathrm{rud}} = [F_N \cdot \cos(-\delta r) - F_A \cdot \sin(-\delta r)] \cdot dx_{\mathrm{rud}}$$

Propeller:
x-direction force F_X:

$$F_X = C_{43} \cdot \chi_P + C_{44} \cdot \chi_P^2, \qquad Y = C_{45} \cdot \chi_P, \qquad L = C_{46} \cdot \chi_P$$

Fuselage:
y-direction force F_Y:

$$F_{Y_{\mathrm{fu}}} = -[C_{39} \cdot k_{39} + C_{40} \cdot k_{40} + C_{41} \cdot k_{41} + C_{42} \cdot k_{42}] \bar{c}_{\mathrm{fu}}$$

Finally, the total force and moment components for the entire aircraft F_X, F_Y, F_Z, L, M, and N are obtained by summing the surface point contribution.

Regression Process

The available measurements of F_X, F_Y, F_Z, L, M, and N each correspond to measured values of linear and angular velocities. These velocities are used to compute the kinematic factors $\boldsymbol{k}$ and $\boldsymbol{h}$, as shown earlier. Thus, each of the m measurements constitutes six equations for the computation of these components. These equations, assembled for the m measurements, become

$$\begin{bmatrix} \begin{Bmatrix} \vdots \\ F_X \\ \vdots \end{Bmatrix} \\ \begin{Bmatrix} \vdots \\ F_Y \\ \vdots \end{Bmatrix} \\ \begin{Bmatrix} \vdots \\ F_Z \\ \vdots \end{Bmatrix} \\ \begin{Bmatrix} \vdots \\ L \\ \vdots \end{Bmatrix} \\ \begin{Bmatrix} \vdots \\ M \\ \vdots \end{Bmatrix} \\ \begin{Bmatrix} \vdots \\ N \\ \vdots \end{Bmatrix} \end{bmatrix}_{6m\times 1} = \begin{bmatrix} \begin{Bmatrix} \vdots & \vdots & \ddots & \vdots & \vdots \\ x_1 & x_2 & \cdots & x_{45} & x_{46} \\ \vdots & \vdots & \ddots & \vdots & \vdots \end{Bmatrix} \\ \begin{Bmatrix} \vdots & \vdots & \ddots & \vdots & \vdots \\ y_1 & y_2 & \cdots & y_{45} & y_{46} \\ \vdots & \vdots & \ddots & \vdots & \vdots \end{Bmatrix} \\ \begin{Bmatrix} \vdots & \vdots & \ddots & \vdots & \vdots \\ z_1 & z_2 & \cdots & z_{45} & z_{46} \\ \vdots & \vdots & \ddots & \vdots & \vdots \end{Bmatrix} \\ \begin{Bmatrix} \vdots & \vdots & \ddots & \vdots & \vdots \\ l_1 & l_2 & \cdots & l_{45} & l_{46} \\ \vdots & \vdots & \ddots & \vdots & \vdots \end{Bmatrix} \\ \begin{Bmatrix} \vdots & \vdots & \ddots & \vdots & \vdots \\ m_1 & m_2 & \cdots & m_{45} & m_{46} \\ \vdots & \vdots & \ddots & \vdots & \vdots \end{Bmatrix} \\ \begin{Bmatrix} \vdots & \vdots & \ddots & \vdots & \vdots \\ n_1 & n_2 & \cdots & n_{45} & n_{46} \\ \vdots & \vdots & \ddots & \vdots & \vdots \end{Bmatrix} \end{bmatrix}_{6m\times 46} \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_{45} \\ C_{46} \end{bmatrix}_{46\times 1}$$

The coefficients x, y, z, l, m, and n in the matrix result from assembling the force and moment contributions of the various points according to the preceding breakdown. They are the result of the kinematic relations giving the local angles of attack and sideslip and the dynamic pressure.

The error is the difference between the vector on the left-hand side and the measurement vector z. The objective is to find $\boldsymbol{C}$, which will minimize the sum of the error squared. $\boldsymbol{C}$ is obtained from

$$\boldsymbol{C} = [\boldsymbol{H}^T\boldsymbol{H}]^{-1}\boldsymbol{H}^T z$$

where $\boldsymbol{H}$ is the $6m \times 46$ matrix of kinematic factors.

An effort was made to eliminate some of these coefficients, which turned out to be insignificant. However, the focus of this paper is on the idea of using the same parameters for both force and moment, and thus the coupling. The model shown is the result of this limited effort.

Results and Discussion

The measurements are available for several spins performed at the NASA Langley Research Center using a light single-engine aircraft.[6] The measurements used were made over a 45-s duration at 0.1-s intervals.

The spins used in this study are labeled G and H in the digital form of the data obtained from NASA Langley Research Center. The regression was performed using the data from both spins G and H. In each case, the coefficients thus obtained were used, along with the control inputs' time histories and the kinematic quantities, to reconstruct the forces

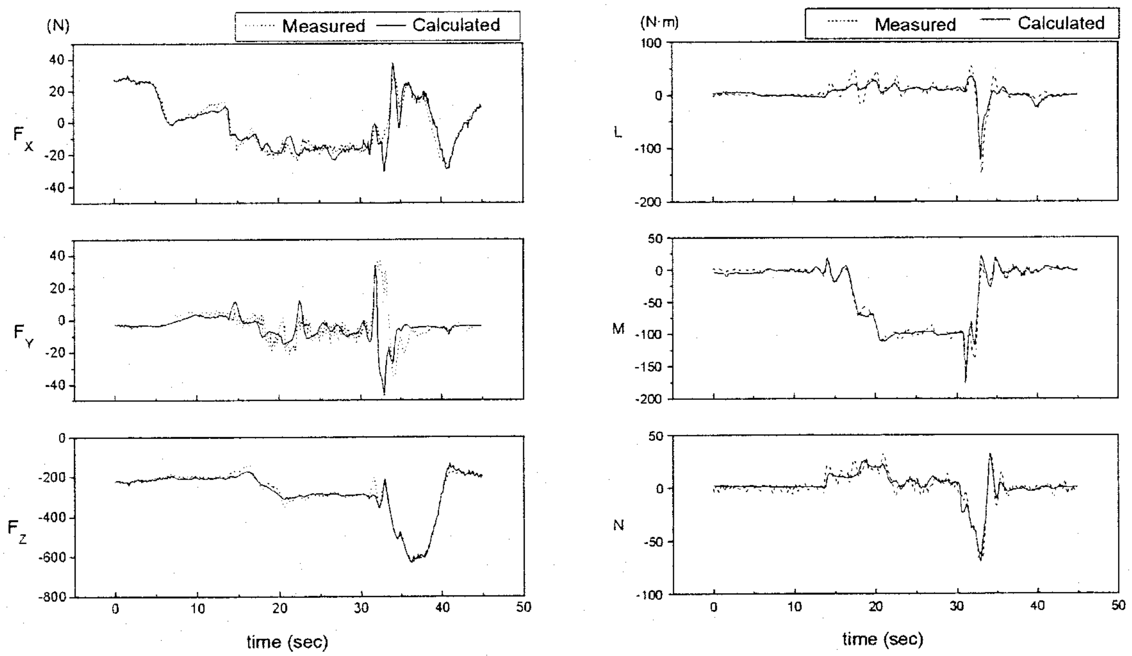

Fig. 1 Comparisons of measured and reconstructed time histories of forces and moments (reconstruction of spin *G* using result with spin *G*).

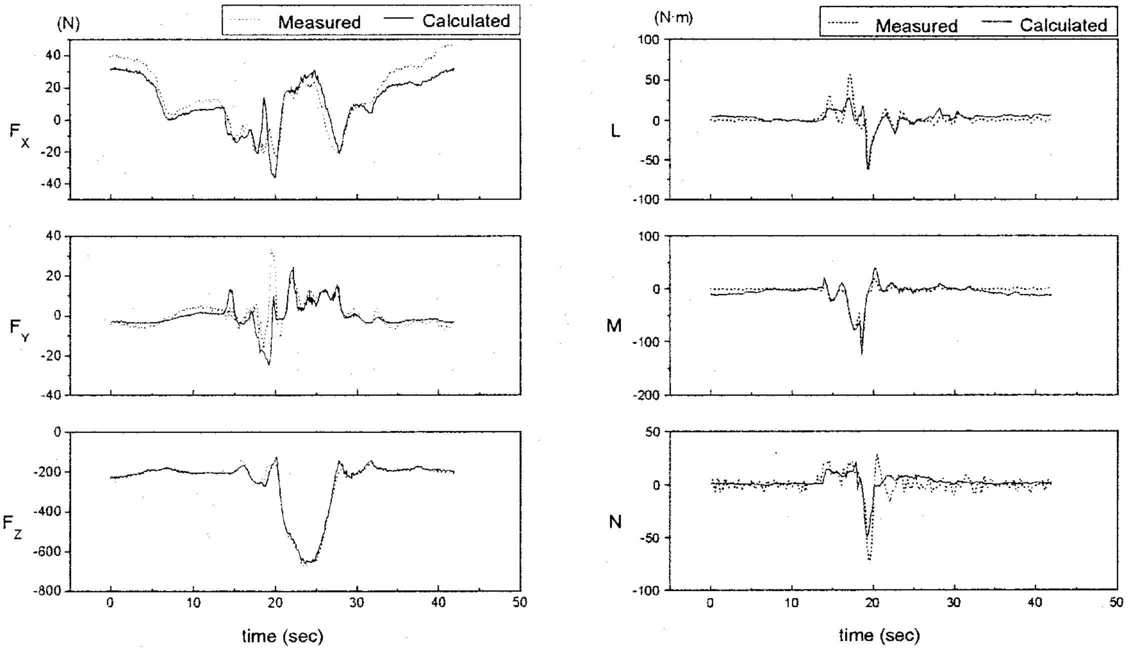

Fig. 2 Comparisons of measured and reconstructed time histories of forces and moments (reconstruction of spin *H* using result with spin *G*).

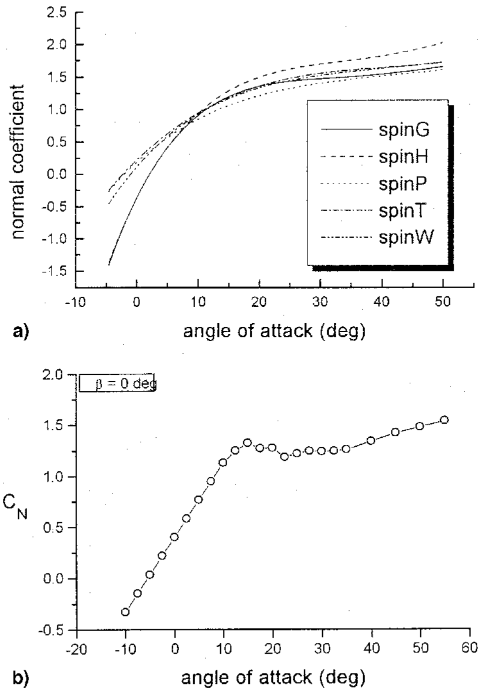

Fig. 3 a) Normal force vs α for wing alone based on estimated parameters, and b) entire aircraft normal force from wind-tunnel tests.

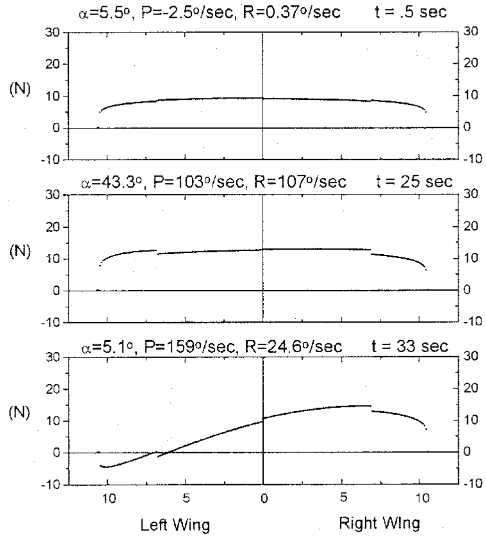

Fig. 4 Normal force distributions on wing.

and moments for spins G and H. In addition, the model from spin G was used to reconstruct the data from spin H. The results of these cases are shown in Figs. 1 and 2. These are time histories of the force and moment measurements along with the reconstructed values from the regression. A reasonable level of robustness is evident in the comparisons, despite the remaining approximation in the model. For example, downwash, sidewash, and propeller wash were not accounted for. The fuselage model is inadequate, and the contribution of the horizontal stabilizer to the rolling moment is neglected. Additionally, plots of the normal force coefficient of the wing vs α were obtained (Fig. 3a). They are comparable in trend and magnitude to the measurements for the entire aircraft as obtained from static tests (Fig. 3b).[7] In Fig. 4, the normal force distribution along the wingspan is plotted for three different conditions to illustrate how the model responds to rotation rates and angle of attack. It must be pointed out, however, that the distribution was forced to be zero at the tips by applying

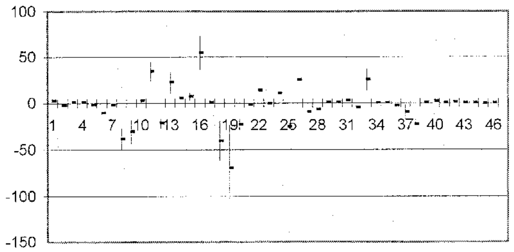

Fig. 5 Parameter mean values and standard deviations.

a superelliptical scaling factor. At this time, there is no other mechanism to achieve this aspect. More examples and more complete treatment of this problem are documented in Ref. 8. Finally, the parameter values and their variances are shown graphically in Fig. 5. One should note that parameters appearing to have very small values are not necessarily insignificant, as they are likely multiplied by large kinematic quantities.

Conclusions

Current parameter estimation techniques for aerodynamic forces and moments are based on estimating the forces and their moments separately. While it is true that the moment equations form an independent set from the force equations, it must be kept in mind that it is the same pressure and shear stress distributions that generate both the forces and their moments. Therefore, the same coefficients described in this paper are estimated by using the force and moment equations simultaneously. This is made possible by the use of the multipoint model and its application to the various aircraft surface components. The preceding results indicate that this approach is a viable alternative to current models used in parameter estimation, when nonlinear flow conditions with large excursions in the vehicle motion variables exist. It was shown that estimation from one set of measurements successfully reproduced those from another flight.

Another advantage of this model is the separation of the force distributions on the various surface elements, thus allowing the analysis of their behavior. For example, the elevator control power deficit during a high-angle-of-attack maneuver can be assessed.

The approach presented in this paper has the potential to produce a reliable and versatile model for aerodynamic performance analysis for use in simulation and control design in this nonlinear regime.

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